what precedes and regulate it, even as the end perfects those things pertaining to it, and they arrange and open the way to what follows. This I now intend to intimate through authority and reason; and in the first place I intend to do so in the human sciences and in the matters of this world, and then in divine knowledge, and lastly according as they are related to the Church and the other three purposes.

CHAPTER II

In which it is proved by authority that every
science requires mathematics.

As regards authority I so proceed. Boetius says in the second prologue to his Arithmetic, "If an inquirer lacks the four parts of mathematics, he has very little ability to discover truth." And again, "Without this theory no one can have a correct insight into truth." And he says also, "I warn the man who spurns these paths of knowledge that he cannot philosophize correctly." And again, "It is clear that whosoever passes these by, has lost the knowledge of all learning." He confirms this by the opinion of all men of weight saying, "Among all the men of influence in the past, who have flourished under the leadership of Pythagoras with a finer mental grasp, it is an evident fact that no one reaches the summit of perfection in philosophical studies, unless he examines the noble quality of such wisdom with the help of the so-called quadrivium." And in particular Ptolemy and Boetius himself are illustrations of this fact. For since there are three essential parts of philosophy, as Aristotle says in the sixth book of the Metaphysics, mathematical, natural, and divine, the mathematical is of no small importance in grasping the knowledge of the other two parts, as Ptolemy teaches in the first chapter of the Almagest, which statement he also explains further in that place. And since the divine part is twofold, as is clear from the first book of the Metaphysics, namely, the first philosophy, which shows that God exists, whose exalted properties it investigates, and civil science, which determines divine worship, and explains many matters concerning God as far as man can receive them. Ptolemy likewise asserts and declares that mathematics is potent

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in regard to both of these branches. Hence Boetius asserts at the end of his Arithmetic that the mathematical means are discovered in civil polity. For he says that an arithmetic mean is comparable to a state that is ruled by a few, for this reason, that in its lesser terms is the greater proportion; but he states that there is a harmonic mean in an aristocratic state for the reason that in the greater terms the greater proportionality is found. The geometrical mean is comparable to a democratic state equalized in some manner; for whether in their lesser or greater terms they are composed of an equal proportion of all. For there is among all a certain parity of mean preserving a law of equality in their relations. Aristotle and his expositors teach in the morals in many places that a state cannot be ruled without these means. Concerning these means an exposition will be given with an application to divine truths. Since all the essential parts of philosophy, which are more than forty sciences distinct in their turn, may be reduced to these three, it suffices now that the value of mathematics has been established by the authorities mentioned.

Now the accidental parts of philosophy are grammar and logic. Alpharabius makes it clear in his book on the sciences that grammar and logic cannot be known without mathematics. For although grammar furnishes children with the facts relating to speech and its properties in prose, meter, and rhythm, nevertheless it does so in a puerile way by means of statement and not through causes or reasons. For it is the function of another science to give the reasons for these things, namely, of that science, which must consider fully the nature of tones, and this alone is music, of which there are numerous varieties and parts. For one deals with prose, a second with meter, a third with rhythm, and a fourth with music in singing. And besides these it has more parts. The part dealing with prose teaches the reasons for all elevations of the voice in prose, as regards differences of accents and as regards colons, commas, periods, and the like. The metrical part teaches all the reasons and causes for feet and meters. The part on rhythm teaches about every modulation and sweet relation in rhythms, because all those are certain kinds of singing, although not so treated as in ordinary singing. For it is called "accent" since it is, as it were, song [*accantus*] from *accino, accinis*. Hence these subjects pertain

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to music as Cassiodorus teaches in music, and Censorinus in his book on Accent, and so too in those on other topics. Authorities on music bear witness to this fact as well as do their books on that science. And Alpharabius agrees with them in his book on the Division of the Sciences. Therefore grammar depends causatively on music.

In the same way logic. For the purpose of logic is the composition of arguments that stir the active intellect to faith and to a love of virtue and future felicity, as we have already shown, which arguments are handed down in the books of Aristotle on these arguments, as has been stated. But these arguments must have a maximum amount of beauty, so that the mind of man may be drawn to the truths of salvation suddenly and without previous consideration, as we are taught in those books. And Alpharabius especially teaches this in regard to the poetic argument, the statements of which should be sublime and beautiful, and therefore accompanied with notable adornment in prose, meter, and rhythm, as befits place, time, personages, and subject for which the plea is made. And thus Aristotle taught in his book on the Poetic Argument, which Hermannus did not venture to translate into Latin on account of the difficulty of the meters, which he did not understand, as he himself states in the prologue to the commentary of Averroës on that book. And therefore the end of logic depends upon music. But the end of everything is its noblest part in every matter and imposes necessity on what is related to it, as Aristotle states in the second book of the Physics; nor have those things any utility of their own which are naturally formed for the end, except when they are related to their end, as is clear in individual cases. And therefore the whole utility of logic is drawn from the relation of all logical arguments to arguments of this kind, and therefore since they depend on the arguments of music, necessarily logic must depend on the power of music. All these facts are in accordance with the opinion of Alpharabius in his book on the Sciences, and they are likewise clearly stated by Aristotle and Averroës in their books, although these are not used by the Latins. But not only does a knowledge of logic depend on mathematics because of its end, but because of its middle and heart, which is the book of Posterior Analytics, for that book teaches the art of demonstration.

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But neither can the fundamental principles of demonstration, nor conclusions, nor the subject as a whole be learned or made clear except in the realm of mathematics, because there alone is there true and forceful demonstration, as all know and as we shall explain later. Therefore of necessity logic depends on mathematics.

What has been said is applicable likewise because of its beginning and not only because of its middle and end. For the book of Categories is the first book of logic according to Aristotle. But it is clear that the category of quantity cannot be known without mathematics. For the knowledge of quantity belongs to mathematics alone. Connected with quantity are the categories of when and where. For when has to do with time, and where arises from place. The category of habit cannot be known without the category of place, as Averroës teaches in the fifth book of the Metaphysics. But the greater part of the category of quality contains the attributes and properties of quantities, because all things that are in the fourth class of quality are called qualities in quantities. And all the attributes of these which are absolutely essential to them are qualities, with which a large part of geometry and arithmetic is concerned, such as straight and curved and other essential qualities of the line, and triangularity and other figures belonging to surface or to a solid body; and the prime and non-factorable in numbers, as Aristotle teaches in the fifth book of the Metaphysics, as well as other essential attributes of numbers. Moreover, whatever is worthy of consideration in the category of relation is the property of quantity, such as proportions and proportionalities, and geometrical, arithmetical, and harmonic means, and the kinds of greater and lesser inequality. Moreover, spiritual substances are known by philosophy only through the medium of the corporeal, and especially the heavenly bodies, as Aristotle teaches in the eleventh book of the Metaphysics. Nor are inferior things known except through superior ones, because the heavenly bodies are the causes of things that are lower. But the heavenly bodies are known only through quantity, as is clear from astronomy. Therefore all the categories depend on a knowledge of quantity of which mathematics treats, and therefore the whole excellence of logic depends on mathematics.